Chapter 3 Logical Reasoning and Methods of Proof

3-1 Implications and Proofs

Synthetic Proof- a proof built using a system of postulates and theorems in which the properties of figures are studied (*not the actual measurements*)

Implication- if-then statement (If A, then B.)

Example) if a piece of fruit is an orange, then it is round.

p🡪q “p implies q” (If p is true, then q is true.)

Converse- If q, then p.

Example) if a piece of fruit is round, then it is orange.

Not necessarily true.

Inverse- If not p, then not q.

Example) if a piece of fruit is not an orange, then it is not round.

Not necessarily true.

Contrapostive- if not q, then not p.

Example) if a piece of fruit is not round, then it is not an orange.

\*\*\*The contrapostive is the logical equivalent to an implication.

Example) given a quadrilateral ABCD with sides AD and BC parallel and congruent. To prove that ABCD is a parallelogram.

|  |  |
| --- | --- |
| Statements | Justifications |
| AD=BC; AD parallel to BC | Given |
| Draw/ construct diagonal AC | Two points determine a line |
| m<BCA=m<DAC | If two parallel lines are cut by a transversal, the alt. int. <’s are congruent |
| AC=AC | Reflexive Property |
| ∆ACD=∆CAB | SAS |
| m<BAC=m<DCA | Corresponding parts of congruent triangles are equal (CPCTE) |
| AB parallel to CD | Alt. int. <’s are congruent, then the lines are parallel |
| ABCD is a parallelogram | Definition parallelogram |

Properties of Quadrilaterals

A quadrilateral is simply a four-sided polygon.

* Sum of the interior angles is 360⁰

A trapezoid is a quadrilateral with at *least* (we’ll talk about it more in 3-3) one pair of parallel sides.

* Isosceles if the legs are congruent

A kite is a quadrilateral with two pairs of consecutive, congruent sides.

* Diagonals are perpendicular

A parallelogram is a quadrilateral with two pairs of parallel sides.

* Opposite sides are congruent.
* Opposite angles are congruent.
* Diagonals bisect each other. (Bisect-divides into two congruent sections.)
* Consecutive angles are supplementary.

A rhombus is a parallelogram with a four congruent sides.

* Diagonals are perpendicular bisectors of each other. (Perpendicular bisectors-90⁰ angles and bisect.)

A rectangle is a parallelogram with four congruent, right angles.

* Diagonals are congruent.

A square is a regular parallelogram (four congruent sides; four congruent angles.)

* All of the above!

All squares are rectangles, rhombuses, and parallelograms.

As an implication…if a quadrilateral is a square, then it is a (rectangle/rhombus/parallelogram). In this case, the converse is not necessarily true! (if a quadrilateral is a (rectangle/rhombus/parallelogram), then it is a square.)

A rhombus is a kite. A kite is not necessarily a rhombus.

3-2 Coordinate vs. Synthetic Proofs

Coordinate Proof: a proof based on a coordinate system in which all points are represented by ordered pairs

Distance Formula:

Midpoint Formula: 

Median of a Triangle: a segment that joins a vertex to the midpoint of the opposite side

Altitude of a Triangle: a perpendicular segment joining a vertex to the opposite side

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3-3 Another Look at Definitions

Trapezoid- a quadrilateral with one pair of parallel sides (traditional)

Trapezoid a quadrilateral with AT LEAST one pair of parallel sides (expanded)

Inclusive Definition­- interpreted broadly

Exclusive Definition- restrictive (narrower)

Ex) A soccer club is organized into U14, U16, and U18 leagues according to the age of their players. Will a 15 year-old be allowed to play in a U18 league?

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Ex2) Tell whether the word “one” means “exactly one” or “at least one?”

* A) There is one line segment connecting any two different points.

Exactly one

* B) The equation has one solution.

At least one

* C) To be considered for the job, an applicant must have one year of experience.

At least one

* D) Each test taker should get one copy of the SAT.

Exactly one

Ex3) True or False

* If a quadrilateral is a rectangle, then it is a parallelogram.

True

* If a quadrilateral is a square, then it is an isosceles trapezoid.

True

* If a quadrilateral is a rhombus, then it is not an isosceles trapezoid.

False

* The diagonals of an isosceles trapezoid never bisect each other.

False

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3-4 Angle in Polygons

The sum of the measures of the angles of a triangle is 180⁰

|  |  |  |
| --- | --- | --- |
| **Polygon** | **Number of Sides** | **S(n)** |
| Triangle | 3 | 180⁰ |
| Quadrilateral | 4 | 360⁰ |
| Pentagon | 5 | 540⁰ |
| Hexagon | 6 | 720⁰ |
| Heptagon | 7 | 900⁰ |
| Octagon | 8 | 1080⁰ |

S(n)= sum of the interior angle measures of a polygon.

S(n)= 180(n-2) where *n* is the number of sides

Regular Polygon- any polygon that has all congruent sides and angles

\*\*The sum of the measures of the exterior angles (one at each vertex) for any convex polygon is 360⁰.\*\*

Ex) Find the measure of each interior and exterior angle of a regular octagon.

S(n)= 180(n-2) S(8)= 180(8-2)= 1080

1080/8=135⁰

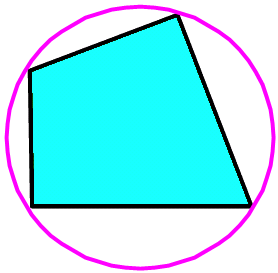
360/8=45⁰ -or- 180-135=45⁰

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3-5 Inscribed Polygons

Inscribed Polygons- Polygon that is enclosed by a circle/polygon so that each vertex lies on the circle/ polygon

Circumscribed- enclosing a figure so that each vertex of the inner figure lies on the enclosing figure



Quadrilateral PQRS is inscribed in the circle

The circle is circumscribed about quadrilateral PQRS.

Circle Vocab

Chord- a line segment joining two points on a circle

Central Angle- an angle with its vertex at the center of the circle

Minor Arc- an arc with a measure less than 180⁰

* Named with just the endpoints of the arc

Semi-circle- an arc measuring 180⁰

* Named with three points on the circle

Major Arc- an arc measuring more than 180⁰ but less than 360⁰

* Named with three points on the circle

Arc- simply part of a circle

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The measure of a central angle is equal to the measure of its intercepted arc.

The measure of an inscribed angle is half of the measure of its intercepted arc.

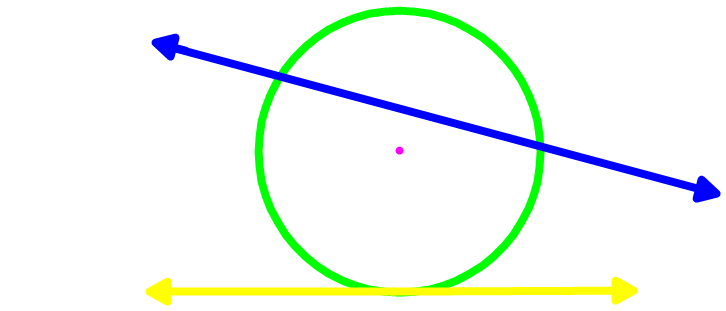
Inscribed Angle- angle formed by two chords that intersect at a point on the circle.

Intercepted Arc- the arc that lies within an inscribed angle.

Theorem: An inscribed angle with an intercepted arc that is a semi-circle must be a right angle. (See above)

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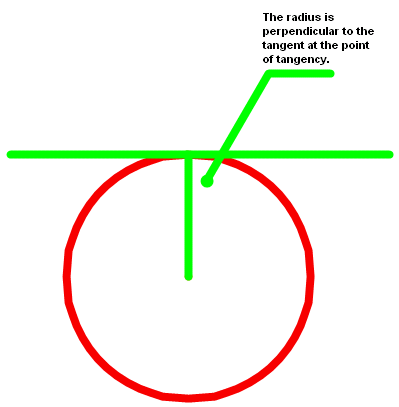
3-7 Indirect Proofs ((Just the name of the section! Focus is all circles!))



Tangent: a line in the plane of a circle that intersects the circle in exactly one point (that point of intersection is called the *point of tangency*.) (Yellow line)

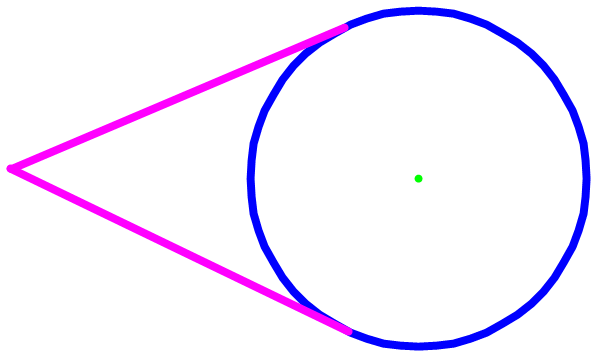
Secant: a line that intersects a circle in two points (Blue)

Theorem: If a line is tangent to a circle, then the line is perpendicular to the radius at the point of tangency.



Theorem: (Converse) If a line in the plane of a circle is perpendicular to a radius at its outer endpoint, then the line is a tangent.

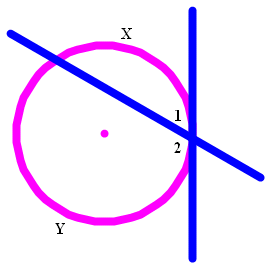
Theorem: If two tangent segments are drawn from the same point to the same circle then they are equal in measure.

The purple segments would have to have the same measure.

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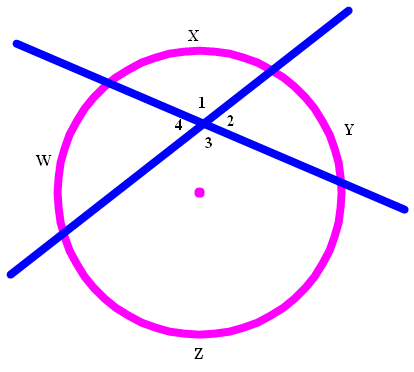
***Theorems with secants and the angles formed:***

Theorem: If a secant and a tangent intersect at the point of tangency, then the measure of each angle formed is one half the measure of its measured arc.



Ex) m<1 = ½ of arc x

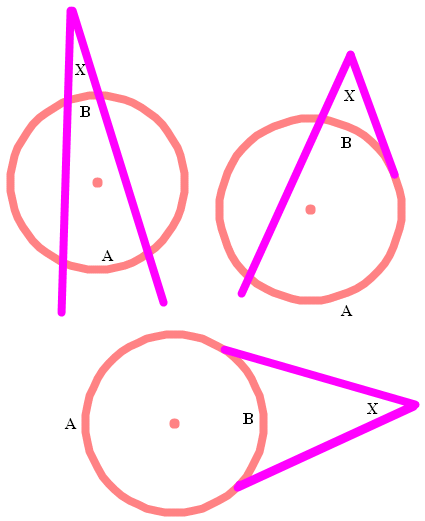
M<2 = ½ of arc y

Theorem: If two secants intersect in the interior of a circle, then the measure of the angle formed is one half the sum of the measures of the arcs intercepted by the angle and its vertical angle.

m<1 & m<3 = ½ of arcs x+z

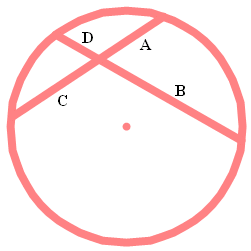
m<2 & m<4 = ½ of arcs w+y

Theorem: If two secants, a secant and a tangent, or two tangents intersect in the exterior of a circle, then the measure of the angle formed is one half the positive difference (absolute value) of the intercepted arcs.

m<x = ½ (a-b)

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**3-7** **Special Segments in a Circle**

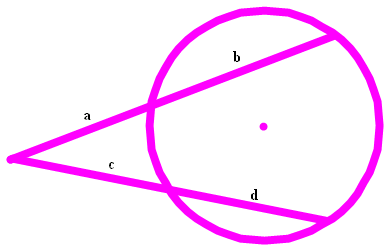
Theorem: If two chords intersect in a circle, then the products of the measures of the chords are equal.

Ex) ac= bd

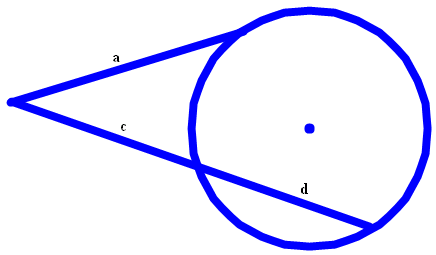
Ex) a= 3 b= 6 c= 4 d= 2

3(4)= 6(2)

12 = 12

Theorem: If two secants segments are drawn to a circle form an external point, then the product of the external segments and the entire secant segments must be equal.

Ex) (a+b)a=(c+d)c

Theorem: If a tangent and a secant are drawn to a circle from a external point, then the square of the measure of the tangent segment is equal to the product of the external segment and the entire secant segment.

Ex) 